Exercise (10.1)

Question 1:

Fill in the blanks

(i) The centre of a circle lies in __________ of the circle. (exterior/interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/interior)

(iii) The longest chord of a circle is a __________ of the circle.

(iv) An arc is a __________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and __________ of the circle.

(vi) A circle divides the plane, on which it lies, in __________ parts.

Solution 1:

(i) The centre of a circle lies in <u>interior</u> of the circle.

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in <u>exterior</u> of the circle.

(iii) The longest chord of a circle is a <u>diameter</u> of the circle.

(iv) An arc is a <u>semi-circle</u> when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and <u>chord</u> of the circle.

(vi) A circle divides the plane, on which it lies, in <u>three</u> parts.

Question 2:

Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Solution 2:

(i) **True**.

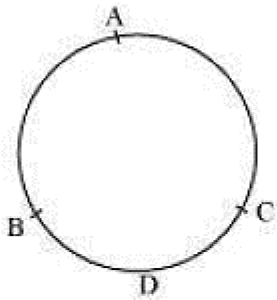
All the points on the circle are at equal distances from the centre of the circle, and this equal distance is called as radius of the circle.

(ii) **False**

There are infinite points on a circle. Therefore, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

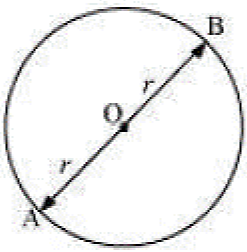
(iii) **False**

Consider three arcs of same length as AB, BC, and CA. It can be observed that for minor arc BDC, CAB is a major arc. Therefore, AB, BC, and CA are minor arcs of the circle.



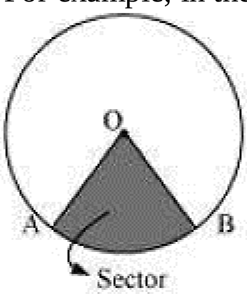
(iv) **True**.

Let AB be a chord which is twice as long as its radius. It can be observed that in this situation, our chord will be passing through the centre of the circle.Therefore, it will be the diameter of the circle.



(v) **False**.

Sector is the region between an arc and two radii joining the centre to the end points of the arc. For example, in the given figure, OAB is the sector of the circle.



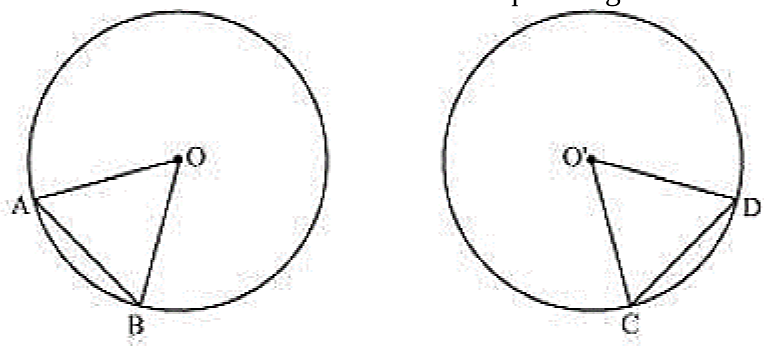
(vi) **True**.

A circle is a two-dimensional figure and it can also be referred to as a plane figure.

Exercise (10.2)

Question 1:
Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution 1:
A circle is a collection of points which are equidistant from a fixed point. This fixed point is called as the centre of the circle and this equal distance is called as radius of the circle. And thus, the shape of a circle depends on its radius. Therefore, it can be observed that if we try to superimpose two circles of equal radius, then both circles will cover each other. Therefore, two circles are congruent if they have equal radius. Consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths.



In ΔAOB and $\Delta CO'D$,
$AB = CD$ (Chords of same length)
$OA = O'C$ (Radii of congruent circles)
$OB = O'D$ (Radii of congruent circles)
$\therefore \Delta AOB \cong \Delta CO'D$ (SSS congruence rule)
$\Rightarrow \angle AOB = \angle CO'D$ (By CPCT)

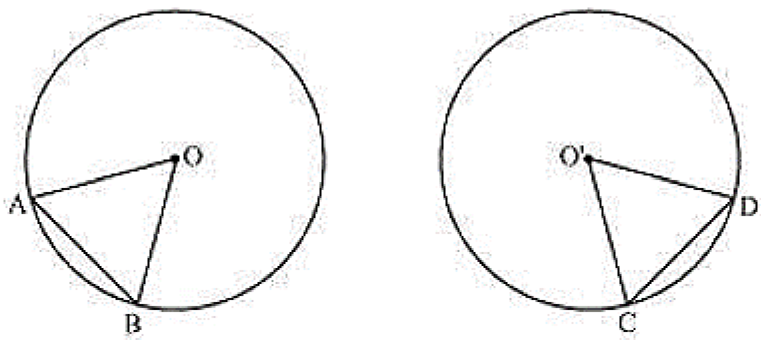
Hence, equal chords of congruent circles subtend equal angles at their centres.

Question 2:

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution 2:

Let us consider two congruent circles (circles of same radius) with centres as O and O'.



In ΔAOB and $\Delta CO'D$,

$\angle AOB = \angle CO'D$ (Given)

$OA = O'C$ (Radii of congruent circles)

$OB = O'D$ (Radii of congruent circles)

$\therefore \Delta AOB \cong \Delta CO'D$ (SSS congruence rule)

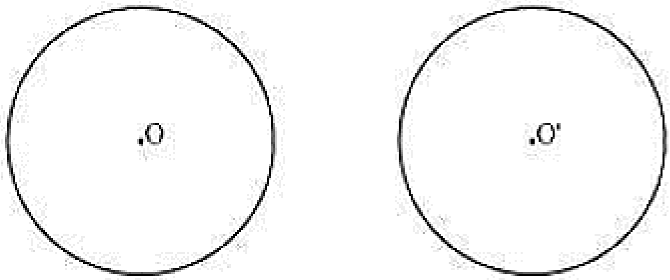
$\Rightarrow AB = CD$ (By CPCT)

Hence, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

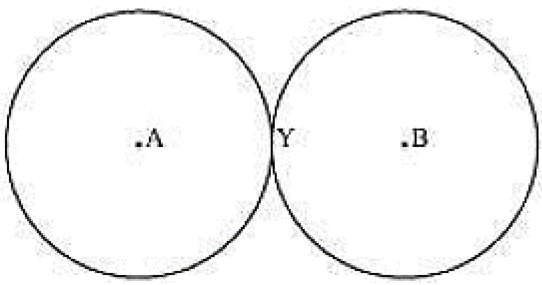
Exercise (10.3)

Question 1:
Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

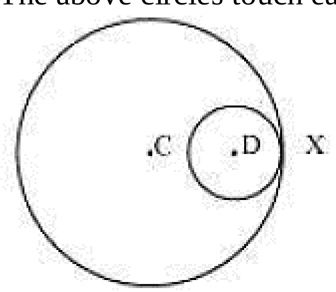
Solution 1:
Consider the following pair of circles.



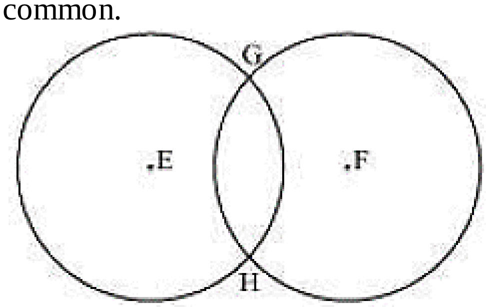
The above circles do not intersect each other at any point. Therefore, they do not have any point in common.



The above circles touch each other only at one point Y. Therefore, there is 1 point in common.



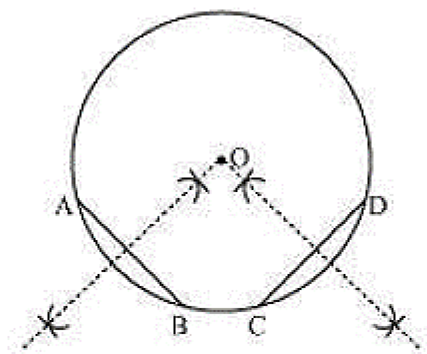
The above circles touch each other at 1 point X only. Therefore, the circles have 1 point in common.



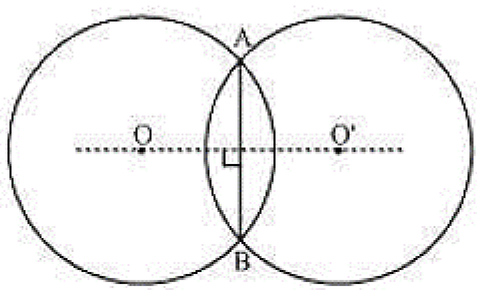
These circles intersect each other at two points G and H. Therefore, the circles have two points in common. It can be observed that there can be a maximum of 2 points in common. Consider the situation in which two congruent circles are superimposed on each other. This situation can be referred to as if we are drawing the circle two times.

Question 2:
Suppose you are given a circle. Give a construction to find its centre.

Solution 2:
The below given steps will be followed to find the centre of the given circle.
***Step* 1**: Take the given circle.
***Step* 2**: Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.
***Step* 3**: Let these perpendicular bisectors meet at point O. Hence, O is the centre of the given circle.



Question 3:
If two circles intersect at two points, then prove that their centres lie on the perpendicular bisector of the common chord.



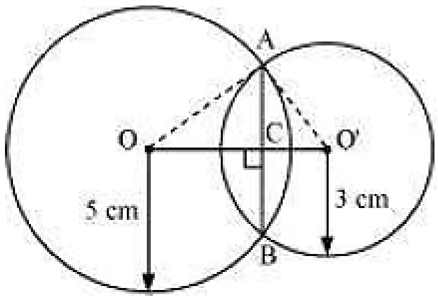
Solution 3:
Consider two circles centered at point O and O', intersecting each other at point A and B respectively.
Join AB. AB is the chord of the circle centered at O. Therefore, perpendicular bisector of AB will pass through O.
Again, AB is also the chord of the circle centered at O'. Therefore, perpendicular bisector of AB will also pass through O'.
Clearly, the centres of these circles lie on the perpendicular bisector of the common chord.

Exercise (10.4)

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution 1:



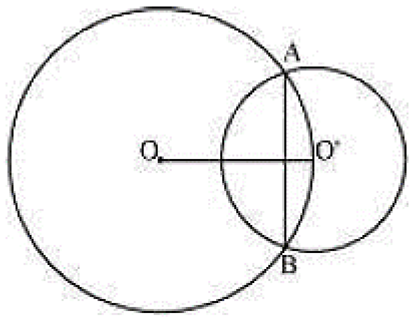
Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively.
OA = OB = 5 cm
O'A = O'B = 3 cm
OO' will be the perpendicular bisector of chord AB.
∴ AC = CB
It is given that, OO' = 4 cm
Let OC be x. Therefore, O'C will be 4 − x.

In ΔOAC,
$OA^2 = AC^2 + OC^2$
$\Rightarrow 5^2 = AC^2 + x^2$
$\Rightarrow 25 - x^2 = AC^2$ ... (1)

In ΔO'AC,
$O'A^2 = AC^2 + O'C^2$
$\Rightarrow 3^2 = AC^2 + (4 - x)^2$
$\Rightarrow 9 = AC^2 + 16 + x^2 - 8x$
$\Rightarrow AC^2 = -x^2 - 7 + 8x$ ... (2)

From Equations (1) and (2), we obtain
$25 - x^2 = -x^2 - 7 + 8x$
$8x = 32$
$x = 4$

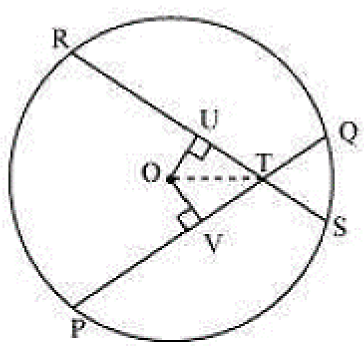
Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle.



$AC^2 = 25 - x^2 = 25 - 42 = 25 - 16 = 9$
$\therefore$ AC = 3 cm
Length of the common chord AB = 2AC = (2 × 3) m = 6 cm

Question 2:
If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution 2:
Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT,
OV = OU (Equal chords of a circle are equidistant from the centre)
∠OVT = ∠OUT (Each 90°)
OT = OT (Common)
$\therefore$ ΔOVT ≅ ΔOUT (RHS congruence rule)
$\therefore$ VT = UT (By CPCT) ... (1)

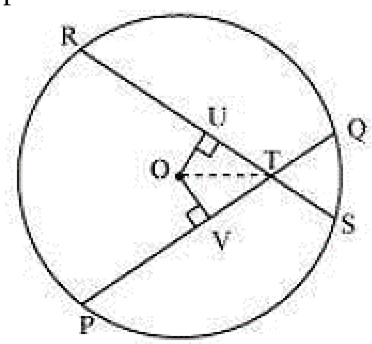
It is given that,
PQ = RS ... (2)
$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$
$\Rightarrow$ PV = RU ... (3)

On adding Equations (1) and (3), we obtain
PV + VT = RU + UT
⇒ PT = RT ... (4)

On subtracting Equation (4) from Equation (2), we obtain
PQ − PT = RS − RT
⇒ QT = ST ... (5)
Equations (4) and (5) indicate that the corresponding segments of chords PQ and RS are congruent to each other.

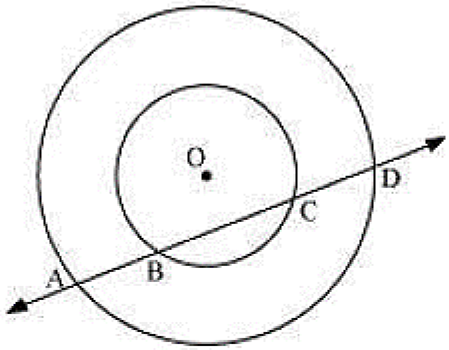
Question 3:
If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution 3:
Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.

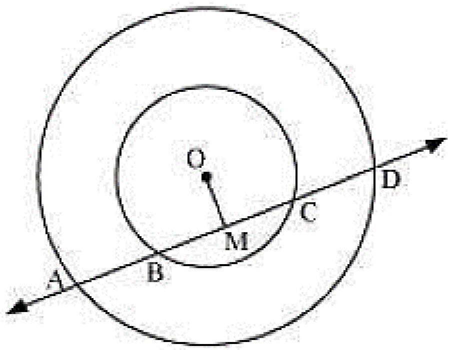


Draw perpendiculars OV and OU on these chords.
In ΔOVT and ΔOUT,
OV = OU (Equal chords of a circle are equidistant from the centre)
∠OVT = ∠OUT (Each 90°)
OT = OT (Common)
∴ ΔOVT ≅ ΔOUT (RHS congruence rule)
∴ ∠OTV = ∠OTU (By CPCT)
Therefore, it is proved that the line joining the point of intersection to the centre makes equal angles with the chords.

Question 4:
If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).



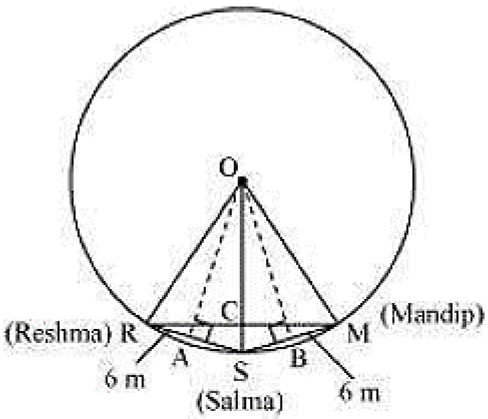
Solution 4:
Let us draw a perpendicular OM on line AD.



It can be observed that BC is the chord of the smaller circle and AD is the chord of the bigger circle.
We know that perpendicular drawn from the centre of the circle bisects the chord.
∴ BM = MC ... (1)
And, AM = MD ... (2)
On subtracting Equation (2) from (1), we obtain
AM − BM = MD − MC
∴ AB = CD

Question 5:
Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Solution 5:
Draw perpendiculars OA and OB on RS and SM respectively.



$AR = AS = \frac{6}{2} = 3$ m

OR = OS = OM = 5 m. (Radii of the circle)

In ΔOAR,

$OA^2 + AR^2 = OR^2$

$OA^2 + (3\text{ m})^2 = (5\text{ m})^2$

$OA^2 = (25 - 9)\text{ m}^2 = 16\text{ m}^2$

OA = 4 m

ORSM will be a kite (OR = OM and RS = SM). We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.

∠RCS will be of 90° and RC = CM

Area of ΔORS = $\frac{1}{2} \times OA \times RS$

$\frac{1}{2} \times RC \times OS = \frac{1}{2} \times 4 \times 6$

$RC \times 5 = 24$

RC = 4.8

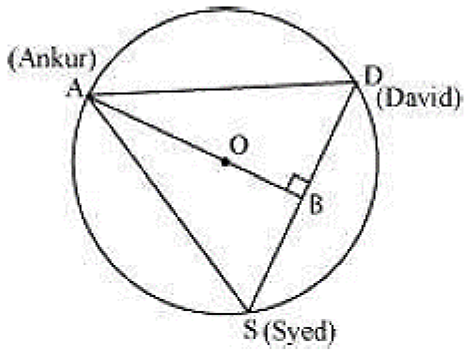
RM = 2RC = 2(4.8) = 9.6

Therefore, the distance between Reshma and Mandip is 9.6 m.

Question 6:

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution 6:



It is given that AS = SD = DA

Therefore, ΔASD is an equilateral triangle.
OA (radius) = 20 m

Medians of equilateral triangle pass through the circumcentre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2:1. As AB is the median of equilateral triangle ASD, we can write

$$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20\text{ m}}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = \left(\frac{20}{2}\right) = 10\text{ m}$$

AB = OA + OB = (20 + 10) m = 30 m
In ΔABD,
$AD^2 = AB^2 + BD^2$

$$AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$$

$$AD^2 = 900 + \frac{1}{4}AD^2$$

$$\frac{3}{4}AD^2 = 900$$

$$AD^2 = 1200$$

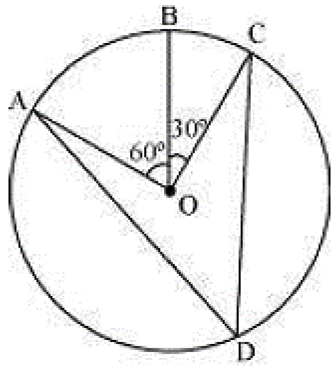
$$AD = 20\sqrt{3}$$

Therefore, the length of the string of each phone will be $20\sqrt{3}$ m.

Exercise (10.5)

Question 1:

In the given figure, A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Solution 1:

It can be observed that

$\angle AOC = \angle AOB + \angle BOC$

$= 60° + 30°$

$= 90°$

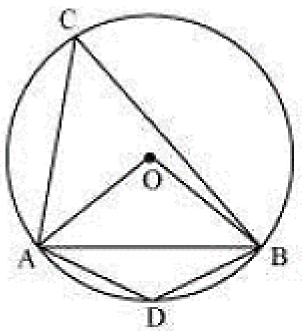
We know that angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(90°) = 45°$$

Question 2:

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution 2:



In ΔOAB,

AB = OA = OB = radius

∴ ΔOAB is an equilateral triangle.

Therefore, each interior angle of this triangle will be of 60°.

∴ ∠AOB = 60°

$$\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}(60°) = 30°$$

In cyclic quadrilateral ACBD,

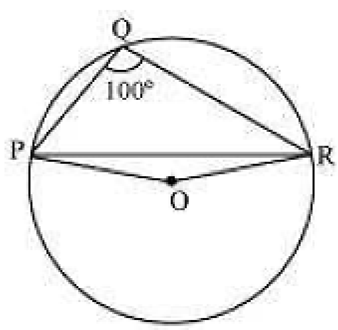
∠ACB + ∠ADB = 180° (Opposite angle in cyclic quadrilateral)

∴ ∠ADB = 180° − 30° = 150°

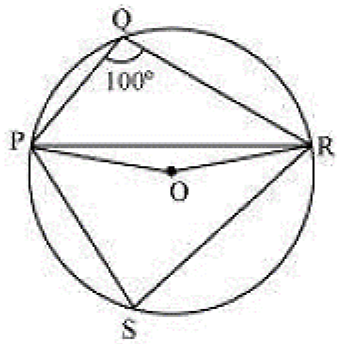
Therefore, angle subtended by this chord at a point on the major arc and the minor arc are 30° and 150° respectively.

Question 3:

In the given figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Solution 3:



Consider PR as a chord of the circle.

Take any point S on the major arc of the circle.

PQRS is a cyclic quadrilateral

$\angle PQR + \angle PSR = 180°$ (Opposite angles of a cyclic quadrilateral)

$\therefore \angle PSR = 180° - 100° = 80°$

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle POR = 2\angle PSR = 2\ (80°) = 160°$

In ΔPOR,

OP = OR (Radii of the same circle)

$\therefore \angle OPR = \angle ORP$ (Angles opposite to equal sides of a triangle)

$\angle OPR + \angle ORP + \angle POR = 180°$ (Angle sum property of a triangle)

$2\angle OPR + 160° = 180°$

$2\angle OPR = 180° - 160° = 20°$

$\angle OPR = 10°$

Question 4:

In fig. 10.38, $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$?

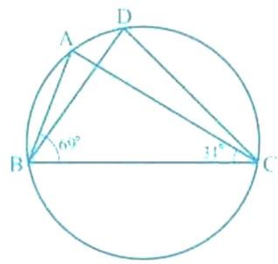


Fig. 10.38

Solution 4:

∠BAC =∠BDC(angles in the same segment of the circle)

In ΔABC,

∠BAC + ∠ABC + ∠ACB =180°(Agnle sum property of a Δ)

Or, ∠BAC+69°+31°=180°

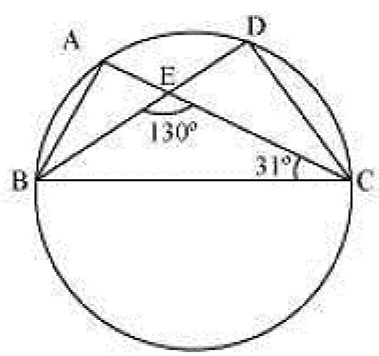
Or, ∠BAC=180°-100°

Or, ∠BAC=80°

Thus, ∠BDC=80°

Question 5:

In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.



Solution 5:

In ΔCDE,

∠CDE + ∠DCE = ∠CEB (Exterior angle)

∴ ∠CDE + 20° = 130°

∴ ∠CDE = 110°

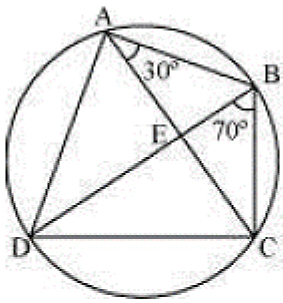
However, ∠BAC = ∠CDE (Angles in the same segment of a circle)

∴ ∠BAC = 110°

Question 6:

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Solution 6:



For chord CD,

∠CBD = ∠CAD (Angles in the same segment)

∠CAD = 70°

∠BAD = ∠BAC + ∠CAD = 30° + 70° = 100°

∠BCD + ∠BAD = 180° (Opposite angles of a cyclic quadrilateral)

∠BCD + 100° = 180°

∠BCD = 80°

In ΔABC,

AB = BC (Given)

∴ ∠BCA = ∠CAB (Angles opposite to equal sides of a triangle)

∴ ∠BCA = 30°

We have, ∠BCD = 80°

∴ ∠BCA + ∠ACD = 80°

$30° + \angle ACD = 80°$

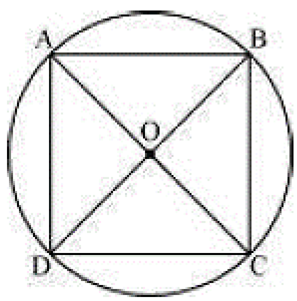
$\therefore \angle ACD = 50°$

$\therefore \angle ECD = 50°$

Question 7:

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution 7:



Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.

$\angle BAD = \frac{1}{2}\angle BOD = \frac{180°}{2} = 90°$ (Consider BD as a chord)

$\angle BCD + \angle BAD = 180°$ (Cyclic quadrilateral)

$\angle BCD = 180° - 90° = 90°$

$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(180°) = 90°$ (Considering AC as a chord)

$\angle ADC + \angle ABC = 180°$ (Cyclic quadrilateral)

$90° + \angle ABC = 180°$

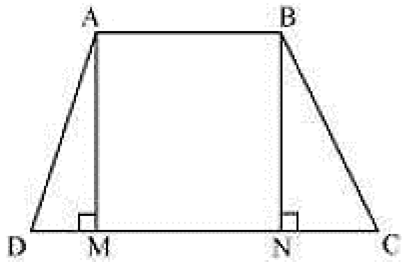
$\angle ABC = 90°$

Each interior angle of a cyclic quadrilateral is of 90°. Hence, it is a rectangle.

Question 8:

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution 8:



Consider a trapezium ABCD with AB || CD and BC = AD.

Draw AM $\perp$ CD and BN $\perp$ CD.

In ΔAMD and ΔBNC,

AD = BC (Given)

∠AMD = ∠BNC (By construction, each is 90°)

AM = BN (Perpendicular distance between two parallel lines is same)

∴ ΔAMD ≅ ΔBNC (RHS congruence rule)

∴ ∠ADC = ∠BCD (CPCT) ... (1)

∠BAD and ∠ADC are on the same side of transversal AD.

∠BAD + ∠ADC = 180° ... (2)

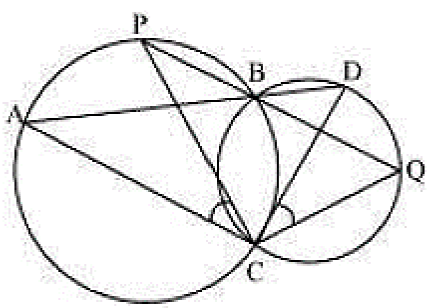
∠BAD + ∠BCD = 180° [Using Equation (1)]

This equation shows that the opposite angles are supplementary.

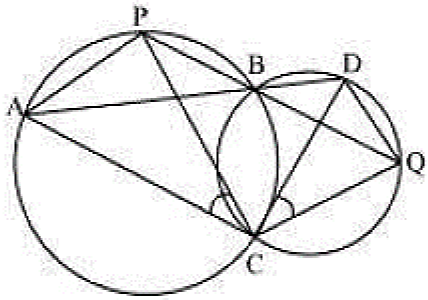
Therefore, ABCD is a cyclic quadrilateral.

Question 9:

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the given figure). Prove that ∠ACP = ∠QCD.



Solution 9:



Join chords AP and DQ.

For chord AP,

∠PBA = ∠ACP (Angles in the same segment) ... (1)

For chord DQ,

∠DBQ = ∠QCD (Angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

∴ ∠PBA = ∠DBQ (Vertically opposite angles) ... (3)

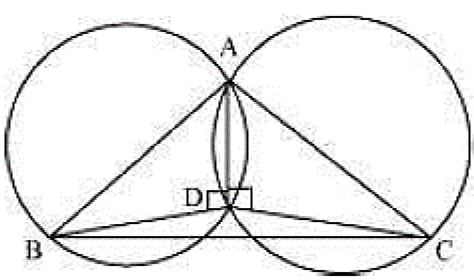
From Equations (1), (2), and (3), we obtain

∠ACP = ∠QCD

Question 10:

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution 10:



Consider a ΔABC.

Two circles are drawn while taking AB and AC as the diameter.

Let they intersect each other at D and let D not lie on BC.

Join AD.

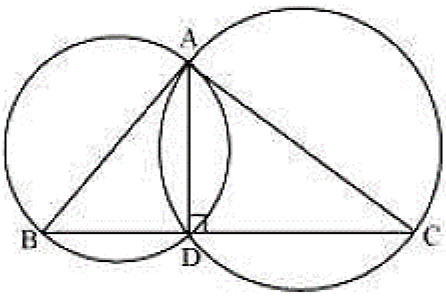
∠ADB = 90° (Angle subtended by semi-circle)

∠ADC = 90° (Angle subtended by semi-circle)

∠BDC = ∠ADB + ∠ADC = 90° + 90° = 180°

Therefore, BDC is a straight line and hence, our assumption was wrong.

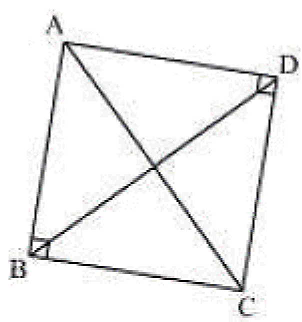
Thus, Point D lies on third side BC of ΔABC.



Question 11:

ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.

Solution 11:



In ΔABC,

∠ABC + ∠BCA + ∠CAB = 180° (Angle sum property of a triangle)

90° + ∠BCA + ∠CAB = 180°

∠BCA + ∠CAB = 90° ... (1)

In ΔADC,

∠CDA + ∠ACD + ∠DAC = 180° (Angle sum property of a triangle)

$90° + \angle ACD + \angle DAC = 180°$

$\angle ACD + \angle DAC = 90°$ ... (2)

Adding Equations (1) and (2), we obtain

$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180°$

$(\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180°$

$\angle BCD + \angle DAB = 180°$ ... (3)

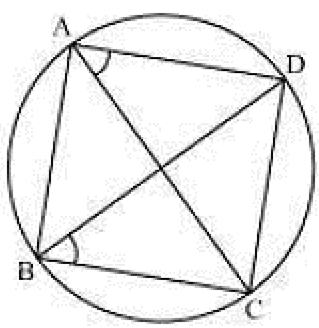
However, it is given that

$\angle B + \angle D = 90° + 90° = 180°$ ... (4)

From Equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

Consider chord CD.

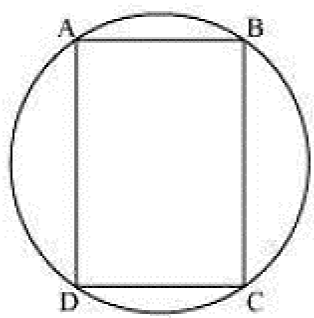
$\angle CAD = \angle CBD$ (Angles in the same segment)



Question 12:

Prove that a cyclic parallelogram is a rectangle.

Solution 12:



Let ABCD be a cyclic parallelogram.

$\angle A + \angle C = 180°$ (Opposite angles of a cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal.

$\angle A = \angle C$ and $\angle B = \angle D$

From Equation (1),

$\angle A + \angle C = 180°$

$\angle A + \angle A = 180°$

$2\angle A = 180°$

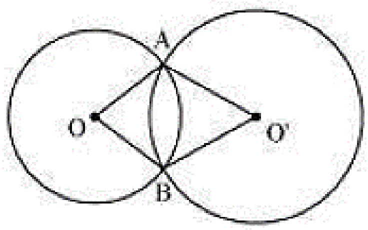
$\angle A = 90°$

Parallelogram ABCD has one of its interior angles as 90°. Therefore, it is a rectangle.

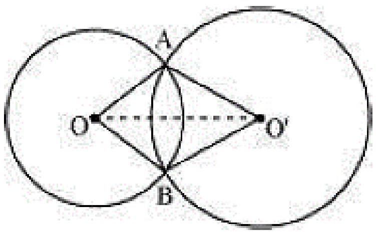
Exercise (10.6)

Question 1:
Prove that line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution 1:



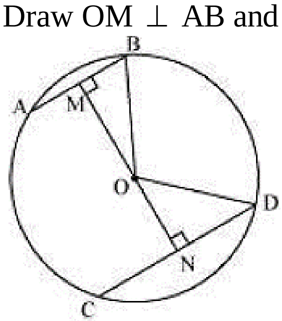
Let two circles having their centres as O and O' intersect each other at point A and B respectively. Let us join O'.



In ΔAOO' and BOO',
OA = OB (Radius of circle 1)
O'A = O'B (Radius of circle 2)
OO' = OO' (Common)
∴ ΔAOO' ≅ ΔBO O' (By SSS congruence rule)
∠OAO' = ∠OBO' (By CPCT)
Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Question 2:
Two chords AB and CD of lengths 5 cm 11cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution 2:
Draw OM ⊥ AB and ON ⊥ CD. Join OB and OD.



BM = $\frac{AB}{2} = \frac{5}{2}$ (Perpendicular from the centre bisects the chord)

ND = $\frac{CD}{2} = \frac{11}{2}$

Let ON be x. Therefore, OM will be 6 − x.

In ΔMOB,

$OM^2 + MB^2 = OB^2$

$$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \qquad \ldots (1)$$

In ΔNOD,

$ON^2 + ND^2 = OD^2$

$$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \qquad \ldots (2)$$

We have OB = OD (Radii of the same circle)

Therefore, from Equations (1) and (2),

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$$

x = 1

From Equation (2),

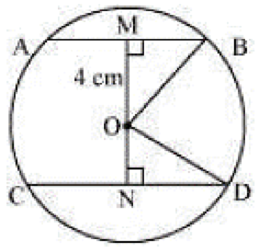
$$(1)^2 + \frac{121}{4} = OD^2$$

$$OD^2 = \frac{121}{4} + 1 = \frac{125}{4}$$

$$OD = \frac{5}{2}\sqrt{5}$$

Therefore, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.=5.6 cm (approx.)

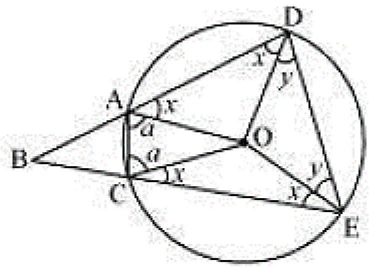
Question 3:
The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution 3:


Let AB and CD be two parallel chords in a circle centered at O. Join OB and OD.
Distance of smaller chord AB from the centre of the circle = 4 cm
OM = 4 cm
MB = $\frac{AB}{2} = \frac{6}{2}$ = 3 cm
In ΔOMB,
$OM^2 + MB^2 = OB^2$
$(4)^2 + (3)^2 = OB^2$
$16 + 9 = OB^2$
OB = $\sqrt{25}$
OB = 5 cm
In ΔOND,
OD = OB = 5 cm (Radii of the same circle)
ND = $\frac{CD}{2} = \frac{8}{2}$ = 4 cm
$ON^2 + ND^2 = OD^2$
$ON^2 + (4)^2 = (5)^2$
$ON^2 = 25 - 16 = 9$
ON = 3 cm
Therefore, the distance of the bigger chord from the centre is 3 cm.

Question 4:
Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution 4:



In ΔAOD and ΔCOE,
OA = OC (Radii of the same circle)
OD = OE (Radii of the same circle)
AD = CE (Given)

$\therefore \Delta AOD \cong \Delta COE$ (SSS congruence rule)

$\angle OAD = \angle OCE$ (By CPCT) ... (1)
$\angle ODA = \angle OEC$ (By CPCT) ... (2)

Also,
$\angle OAD = \angle ODA$ (As OA = OD) ... (3)

From Equations (1), (2), and (3), we obtain
$\angle OAD = \angle OCE = \angle ODA = \angle OEC$
Let $\angle OAD = \angle OCE = \angle ODA = \angle OEC = x$
In ΔOAC,
OA = OC
$\therefore \angle OCA = \angle OAC$ (Let a)

In ΔODE,
OD = OE
$\angle OED = \angle ODE$ (Let y)
ADEC is a cyclic quadrilateral.
$\therefore \angle CAD + \angle DEC = 180°$ (Opposite angles are supplementary)
$x + a + x + y = 180°$
$2x + a + y = 180°$
$y = 180° - 2x - a$ ... (4)

However, $\angle DOE = 180° - 2y$
And, $\angle AOC = 180° - 2a$
$\angle DOE - \angle AOC = 2a - 2y = 2a - 2(180° - 2x - a)$
$= 4a + 4x - 360°$ ... (5)
$\angle BAC + \angle CAD = 180°$ (Linear pair)
$\therefore \angle BAC = 180° - \angle CAD = 180° - (a + x)$
Similarly, $\angle ACB = 180° - (a + x)$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ (Angle sum property of a triangle)

$\angle ABC = 180^\circ - \angle BAC - \angle ACB$

$= 180^\circ - (180^\circ - a - x) - (180^\circ - a - x)$

$= 2a + 2x - 180^\circ$

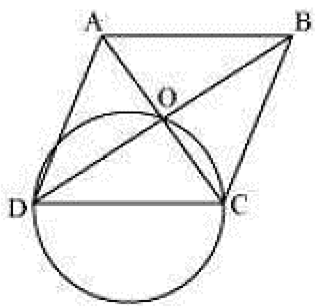
$= \frac{1}{2}[4a + 4x - 360^\circ]$

$\angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$ [Using Equation (5)]

Question 5:

Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

Solution 5:



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

$\therefore \angle COD = 90^\circ$

Also, in rhombus, the diagonals intersect each other at 90°.

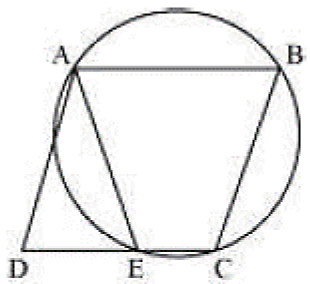
$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$

Clearly, point O has to lie on the circle.

Question 6:

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

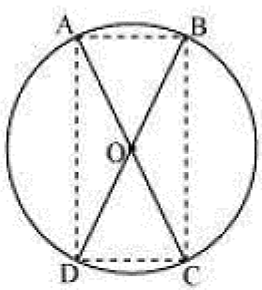
Solution 6:



It can be observed that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of the opposite angles is 180°.

$\angle AEC + \angle CBA = 180°$
$\angle AEC + \angle AED = 180°$ (Linear pair)
$\angle AED = \angle CBA$ ... (1)
For a parallelogram, opposite angles are equal.
$\angle ADE = \angle CBA$ ... (2)
From (1) and (2),
$\angle AED = \angle ADE$
AD = AE (sides opposite to equal Angles of a triangle are equal).

Question 7:
AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Solution 7:


Let two chords AB and CD are intersecting each other at point O.

In ΔAOB and ΔCOD,
OA = OC (Given)
OB = OD (Given)
$\angle AOB = \angle COD$ (Vertically opposite angles)
$\therefore \Delta AOB \cong \Delta COD$ (SAS congruence rule)
AB = CD (By CPCT)

Similarly, it can be proved that $\Delta AOD \cong \Delta COB$

∴ AD = CB (By CPCT)

Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram.

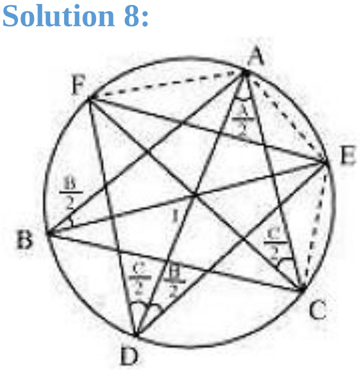
We know that opposite angles of a parallelogram are equal.
∴ ∠A = ∠C

However, ∠A + ∠C = 180° (ABCD is a cyclic quadrilateral)
∠A + ∠A = 180°
2∠A = 180°
∴ ∠A = 90°

As ACBD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.
∠A is the angle subtended by chord BD.
And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

Question 8:
Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$, and $90° - \frac{1}{2}C$.

Solution 8:



It is given that BE is the bisector of ∠B.

∴ ∠ABE = $\frac{\angle B}{2}$

However, ∠ADE = ∠ABE (Angles in the same segment for chord AE)

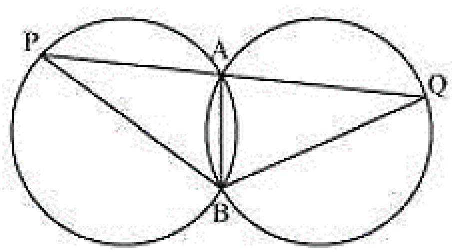
∴ ∠ADE = $\frac{\angle B}{2}$

Similarly, ∠ACF = ∠ADF = $\frac{\angle C}{2}$ (Angle in the same segment for chord AF)

∠D = ∠ADE + ∠ADF

$$= \frac{\angle B}{2} + \frac{\angle C}{2}$$
$$= \frac{1}{2}(\angle B + \angle C)$$
$$= \frac{1}{2}(180° - \angle A)$$
$$= 90° - \frac{1}{2}\angle A$$
Similarly, it can be proved that
$$\angle E = 90° - \frac{1}{2}\angle B$$
$$\angle F = 90° - \frac{1}{2}\angle C$$

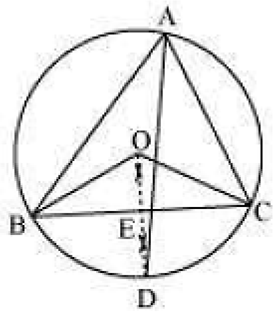
Question 9:
Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution 9:


AB is the common chord in both the congruent circles.
∴ ∠APB = ∠AQB
In ΔBPQ,
∠APB = ∠AQB
∴ BQ = BP   (Sides opposite to equal angles of a triangle are equal)

Question 10:
In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circum-circle of the triangle ABC.

Solution 10:



Let perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D. Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcentre O of the circle. $\angle BOC$ and

$\angle BAC$ are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\angle BOC = 2\angle BAC = 2\angle A$ ... (1)

In ΔBOE and ΔCOE,
OE = OE (Common)
OB = OC (Radii of same circle)
$\angle OEB = \angle OEC$ (Each 90° as OD $\angle$ BC)
$\therefore \Delta BOE \cong \angle COE$ (RHS congruence rule)
$\angle BOE = \angle COE$ (By CPCT) ... (2)

However, $\angle BOE + \angle COE = \angle BOC$
$\therefore \angle BOE + \angle BOE = 2\angle A$ [Using Equations (1) and (2)]
$2\angle BOE = 2\angle A$
$\angle BOE = \angle A$
$\angle BOE = \angle COE = \angle A$

The perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D.
$\therefore \angle BOD = \angle BOE = \angle A$ ... (3)

Since AD is the bisector of angle $\angle A$,
$\angle BAD = \dfrac{\angle A}{2}$
$\therefore 2\angle BAD = \angle A$ ... (4)

From Equations (3) and (4), we obtain
$\angle BOD = 2\angle BAD$

This can be possible only when point BD will be a chord of the circle. For this, the point D lies on the circumcircle.

Therefore, the perpendicular bisector of side BC and the angle bisector of ∠A meet on the circumcircle of triangle ABC.